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The evolution of rotation in the early history of the Solar System

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Theories that require the co-genetic formation of the Sun and planets have difficulty in explaining the slow rotation of the Sun. An analysis is made of various mechanisms for slowing down the core of an evolving nebula. Two of these involve a high magnetic dipole moment for the early Sun. The first envisages magnetic linkage to an external plasma but requires a dipole moment 10^6 times that of the present Sun. The other is based on the co-rotation of matter leaving the Sun during a T Tauri stage, and requires a dipole moment 10^4 times the present value. A mechanical process for transferring angular momentum outward involving dissipation in a solar-nebula disc is incapable of giving what is required. Two processes of star formation in a turbulent cloud are discussed. Both are capable of giving a slowly rotating Sun. Various models for producing planets are examined in relation to the spin they would produce. Planets formed from floccules would be spinning quickly but could evolve in such a way as to give observed spins for giant planets and also satellite families. Accretion models are very sensitive to assumptions, and parameters and can be adjusted to explain almost any observation. Protoplanets formed in elliptical orbits would acquire spin angular momentum through solar tidal action and would evolve to give reasonable spin rates and regular satellite families. The various tilts of their spin axes could be explained by interactions between protoplanets in the early Solar System.

1. Introduction

It is inevitable that any discussion of theories for the origin of the axial spins of the Sun and the planets will include consideration of the orbits and formation of planets and satellites. A major problem that any theory of the origin of the Solar System must address is the partitioning of angular momentum between the Sun and the planets. The Sun contains 99.87 % of the total mass of the system but its spin accounts for only 1 % or so of the total angular momentum; the remainder is in the planetary orbits with more than 60 % in the orbit of Jupiter alone.

On the basis of their measurement of the apparent oblateness of the Sun's disc, Dicke & Goldenberg (1967) have suggested that interior regions of the Sun, below the convecting zone, may be rotating up to 20 times faster than the surface rate. This conclusion is strongly disputed. The resulting mass quadrupole moment would have implications for the advance of the perihelion of Mercury and hence for relativity theory. Nevertheless it should be kept in mind as a possibility that the Sun's share of the angular momentum of the Solar System may be a few times greater than the 1 % previously quoted.

2. The evolution of solar spin

(a) *The solar nebula*

Cosmogenic theory has been dominated for the past decade by the solar-nebula theory (Cameron & Pine 1973; Cameron 1978), although its influence is now waning. This is a modern version of an idea, first advanced by Laplace (1796), whereby a collapsing, rotating nebula spontaneously gave rise to the Sun in a central condensation and planets from a disc of material left behind in the equatorial plane. Although it is generally accepted that the straightforward evolution of a solar nebula will not explain the angular momentum distribution in the solar system, it is instructive, by the use of a simple model, to determine the extent of the problem.

We consider a uniformly rotating disc-shaped nebula of unit radius with areal density given by

$$\rho = \rho_0 \exp(-x^2/\sigma^2). \tag{1}$$

The mass out to a distance d is given by

$$M(d) = \pi\rho_0\sigma^2\{1-\exp(-d^2/\sigma^2)\}. \tag{2}$$

If a fraction, f, of the total mass of the disc is to form the Sun then

$$f\pi\rho_0\sigma^2\{1-\exp(-1/\sigma^2)\} = M_\odot, \tag{3}$$

and the distance d_0 out to which all the disc material will be contained in the Sun is given by

$$\{1-\exp(-d_0^2/\sigma^2)\} = f\{1-\exp(-1/\sigma^2)\}. \tag{4}$$

The angular momentum associated with material out to a distance d_0 will be

$$H = \pi\rho_0\sigma^2\omega[\sigma^2\{1-\exp(-d_0^2/\sigma^2)\}-d_0^2\exp(-d_0^2/\sigma^2)], \tag{5}$$

and as a fraction of the total angular momentum of the disc it is

$$\alpha = \frac{\sigma^2\{1-\exp(-d_0^2/\sigma^2)\}-d_0^2\exp(-d_0^2/\sigma^2)}{\sigma^2\{1-\exp(-1/\sigma^2)\}-\exp(-1/\sigma^2)}. \tag{6}$$

For a particular degree of central condensation of the nebula, given by σ, and for any chosen value of f, it is possible to find d_0 from (4). Then from (6) the value of α may be found. In table 1 values of α are shown for various combinations of σ and f.

TABLE 1. THE PROPORTION OF ANGULAR MOMENTUM IN AN INNER REGION OF THE NEBULA IN TERMS OF f, THE FRACTION OF THE CONTAINED MASS AND σ, THE SPREAD PARAMETER DEFINED IN (1)

$\sigma\backslash f$	0.1	0.2	0.4	0.5	0.6	0.8	0.9
2.0	0.009	0.037	0.152	0.240	0.348	0.629	0.803
1.0	0.007	0.031	0.133	0.213	0.315	0.596	0.780
0.5	0.005	0.023	0.099	0.162	0.246	0.501	0.697
0.25	0.005	0.021	0.094	0.153	0.233	0.478	0.670
0.1	0.005	0.021	0.094	0.153	0.233	0.478	0.670

For any value of f less than 0.9984 some material must be lost from the outer disc for the eventual combined mass of the planets to be correctly related to that of the Sun. Many solar-nebula models postulate a disc with an initial mass much greater than the present mass of the Solar System. If the proportion of angular momentum lost is the same as that of mass lost, then the figures in table 1 indicate that the angular momentum of the central body will be at least 99 % of the total angular momentum. For example with $\sigma = 2$ and $f = 0.2$ the outer region will need to lose 0.9996 of its mass to obtain the right ratio of masses for the Sun and planets. The angular momentum ratio then becomes

$$0.037/0.963\times 0.0004 = 96.1,$$

which corresponds to the Sun possessing 99% of the total angular momentum. For $\sigma = 0.25$ and $f = 0.9$ the corresponding figure is 99.3%.

It is possible that a different form of mass distribution could give a slightly different result but the conclusion is inescapable that a simple solar-nebula model would lead to a central body with angular momentum too great by three or four orders of magnitude. Actually, it would not even be possible to produce the Sun with so much angular momentum since it would be rotationally unstable. In fact, the Sun is a fairly typical late-type star in its slow rate of rotation. There is a considerable variation in the observed equatorial speeds of stars, but there is a considerable drop in speeds for stars later than about F4. In addition, observations of young stellar clusters suggest that spin rates of stars are correlated with age and that young solar-type stars rotate 6–7 times faster then does the Sun. It is clear that mechanisms may exist that can remove angular momentum from an existing star. What we shall now do is to examine a number of possible mechanisms whereby the central body may lose angular momentum during the process of initial collapse.

(*b*) *Braking from magnetic fields*

Alfvén & Arrhenius (1976) suggested a mechanism whereby, through the influence of the Sun's magnetic field, a very high initial solar spin rate may have been reduced. This model takes as its starting point a developing nebula where the central body, destined to form the Sun, was surrounded by outer material within which the planets would form (figure 1). The outer material was in the form of a conducting plasma and this is cut by the lines of force due to the rotation of the solar magnetic field. If the angular speed of the proto-Sun is greater than that of the plasma then currents will be generated in the plasma in the way shown in figure 1. These currents will give $\boldsymbol{i} \times \boldsymbol{B}$ forces in the parts of the circuits ab, cd and c′d′ tending to slow the rotation of the protosun and speed up that of the plasma. The net effect of this process is to transfer angular momentum from the protostar to exterior material.

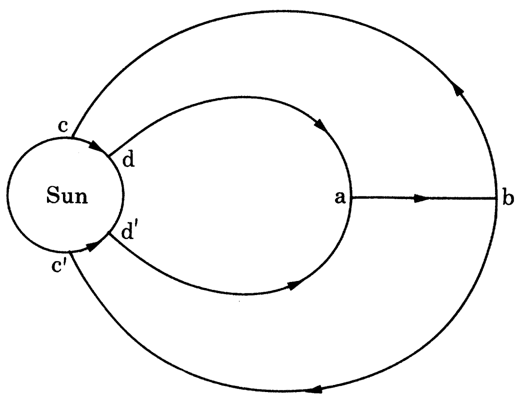

FIGURE 1. The Sun rotating faster than an external plasma gives currents along field lines as shown. Along ab, cd and c′d′ there are $\boldsymbol{i} \times \boldsymbol{B}$ forces slowing down the rotation of the Sun and imparting angular momentum to the plasma.

Alfvén & Arrhenius estimated that, to achieve the required coupling, the magnetic dipole moment would need to have been about 5×10^{28} T m^3; this compares with the present value of about 8×10^{22} T m^3. The requirement for such a high value must give some misgiving and Freeman (1978) asserts that such a value is very unlikely. From a number of different points of view Freeman has estimated a range of possible values of dipole moment for the early Sun between 1×10^{24} and 5×10^{26} T m^3.

Another proposed mechanism for removing solar angular momentum, which also depends on the solar magnetic field, involves mass loss due to a solar wind. Charged particles leaving the Sun will be constrained to move along field lines in the vicinity of the Sun where the field is high and, since the field lines rotate with the Sun, the charged particles will co-rotate with the Sun while they are so constrained.

The condition governing whether the motion of the charged particles will follow field lines or not will depend on the relative strengths of magnetic pressure (energy density).

$$P_B = B^2/2\mu_0, \tag{7}$$

where B is the field and μ_0 the permeability of free space, and gas pressure

$$P_g = nk\theta + nmv^2, \tag{8}$$

where θ is the temperature and n is the number density of particles of mean mass m and bulk flow velocity v. The two terms on the right side of (8) represent the normal thermal pressure plus a term arising from bulk flow. We shall assume that co-rotation of particles with the Sun will break down when the pressures given by (7) and (8) are equal. This will happen at a distance r such that

$$D^2/2\mu_0 r^6 = \dot{M}(v + k\theta/vm)/4\pi r^2, \tag{9}$$

where D is the solar dipole moment and $\dot{M}$ the rate of mass loss from the Sun. For protons with $v = 5\times10^5$ m s^{-1}, a typical solar-wind flow speed, the temperature-dependent term will be negligible unless $\theta \approx 10^7$ K and so it may be ignored. Thus co-rotation will persist to a distance

$$r = (2\pi D^2/\mu_0 \dot{M} v)^{\frac{1}{4}}. \tag{10}$$

With present day values, $\dot{M} = 2\times10^9$ kg s^{-1} and $D = 8\times10^{22}$ T m^3, $r = 3.4\ R_\odot$. Thus each unit mass lost takes with it $(3.4)^2$ as much angular momentum as is possessed by a unit mass at the solar equator. At this rate, in the whole of its life, the Sun would have lost less than 1 % of its original angular momentum.

It is very often assumed that the Sun passed through a T Tauri stage when it was young. Some estimates of the mass lost are that it would be at a rate of $10^{-7}\ M_\odot$ a^{-1} for about 10^6 a. If such a mass loss occurred with a wind speed of 500 km s^{-1} and a dipole moment of 5×10^{26} T m^3 then, from (10), $r = 6.36\ R_\odot$. The rate of change of angular velocity per unit mass loss is given by

$$\mathrm{d}\Omega/\mathrm{d}M = r^2\Omega/\alpha M R_\odot^2, \tag{11}$$

where the radius of the Sun is assumed constant and the moment-of-inertia factor α is about 0.1 for a centrally condensed star. Integrating this expression gives

$$\Omega = \Omega_0 (M/M_0)^{r^2/\alpha R_\odot^2} \tag{12}$$

and with $M/M_0 = 0.9$ this gives

$$\Omega \approx 10^{-19}\,\Omega_0. \tag{13}$$

It is clear that this would be a very effective means of braking the rotation of the early Sun but it is as well to note that the result is very sensitive to the parameters used and to the assumptions made. The rising cool material which is detected from T Tauri stars, and is

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identified as the material being lost, shows strong spectral lines of neutral hydrogen and so will not be strongly ionized as is the present solar wind. Thus the coupling of the lost material to the stellar magnetic field may be quite feeble. Again, if the slow rotation of the Sun is ascribed to a T Tauri phase then does this apply to all the other slowly-rotating late-type stars as well? This would seem improbable, especially for stars much less massive than the Sun for which a T Tauri stage is very unlikely.

(c) *A mechanical process*

A mechanical mechanism whereby angular momentum could be transferred outwards from the centre of a rotating disc nebula has been suggested by Lynden-Bell & Pringle (1974). The case they considered was of a disc rotating with a balance between centrifugal force and gravity. Since this condition would involve differential rotation, and therefore, shear within the viscous material, there would be some energy dissipation within it. Thus the disc must evolve in such a way that its total energy of motion is reduced while its angular momentum remains constant. The pattern of behaviour giving this condition is that matter far from the rotation axis moves outwards, while matter closer to the axis moves inwards, the result being a net transfer of angular momentum to the outer material. Cameron (1978) has appealed to this mechanism to slow down the spin of the core during the evolution of a solar nebula. However, it can be shown that the mechanism, by itself, is quite ineffective in solving the angular momentum problem.

Once an element of mass has entered the central core, then the mechanism will cease to operate. The assumption will be made, favourable to the effectiveness of the process, that at the time of entering the core the density of the material ρ is always the same and equal to the present mean density of the Sun. When the material joins the central core it will be taken to be in Keplerian orbit about the growing central mass. The rate of increase of angular momentum per unit mass of added material is

$$\mathrm{d}H/\mathrm{d}M = (GMr)^{\frac{1}{2}}, \tag{14}$$

where M is the mass and r the radius of the growing core. This can be transformed to

$$\mathrm{d}H/\mathrm{d}M = G^{\frac{1}{2}}(3/4\pi\rho)^{\frac{1}{6}}\,M^{\frac{2}{3}}, \tag{15}$$

which gives by integrating to the present mass of the Sun the final angular momentum

$$H_{\mathrm{f}} = \tfrac{3}{5}G^{\frac{1}{2}}(3/4\pi\rho)^{\frac{1}{6}}\,M_{\odot}^{\frac{5}{3}}. \tag{16}$$

Substituting solar values for ρ and $M_{\odot}$ gives $H_{\mathrm{f}} = 3.7\times10^{44}$ kg m^2 s^{-1} which is more than three orders of magnitude greater than the empirical value of about 2×10^{41} kg m^2 s^{-1}. Tinkering with some of the assumptions cannot make this conclusion much more favourable; the result is not surprising in view of the well known result that one Jupiter mass in orbit about the solar equator has about three times the angular momentum of the present Sun.

The message from this simple analysis is that without some auxiliary process, the Sun could not form at all from a solar nebula unless the solar nebula was rotating so slowly that it could not provide the angular momentum to explain the planetary orbits.

(d) *Other processes of star formation*

Alternative processes for forming solar-type stars with low spin rates have been suggested by McCrea (1960, 1978) and Woolfson (1979). In McCrea's model the behaviour of a cloud of turbulent material, which is eventually to form a stellar cluster, is studied. Turbulence is modelled by floccules, regions of higher than average density moving in a random way with respect to each other (figure 2*a*). When floccules collide they combine and eventually large aggregates will form (figure 2*b*) which, by gravity-assisted further growth, will give rise to a cluster of stars. The parameters of the floccule model were chosen to give a planetary system resembling the Solar System; planets were formed from small floccule aggregates captured by a central star. Some 10^5 floccules were required to form the Sun and, since they came together in a random way so that $\sqrt{N}$ statistics prevailed, there was no systematic tendency for a large angular momentum to be acquired. McCrea's estimate of angular momentum for the Sun, 7×10^{41} kg m^2 s^{-1}, is only three times the value deduced from observation.

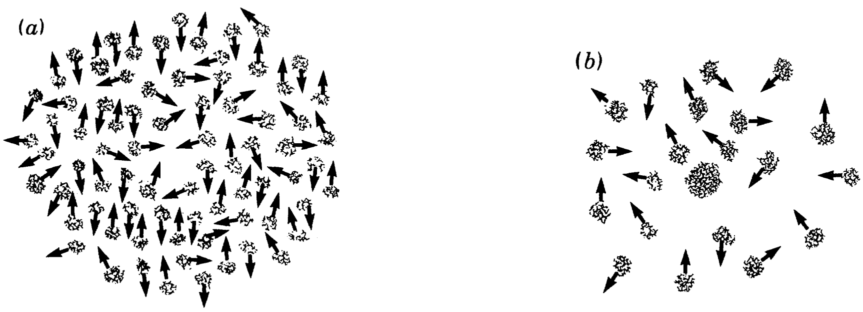

FIGURE 2. (*a*) High-density floccules with random motions in a lower-density background medium. (*b*) The formation of a protostar by aggregation of floccules.

There is a problem with McCrea's model in terms of floccule stability since the lifetime of a floccule, or any small number of them in aggregation, is about one year while the estimated mean time between floccule collisions is around 30 years. Woolfson (1979) has considered a similar model of a turbulent cloud but with much more massive turbulent elements. The collision of two turbulent elements creates a high-density region which quickly cools and may subsequently collapse to form a star (figure 3). Stars of mass less than about 1.4 $M_\odot$ are formed in this way and are shown to rotate fairly slowly (equatorial speeds 3–40 km s^{-1}). On the other hand, more massive stars, which grow by accretion in dense parts of the cloud, are found to have angular momenta related to mass in much the way observed, as illustrated in figure 4.

There are many assumptions in this model – for example that the turbulence scale length is a Jeans length – and it could be that some theoretical objections could be raised to this or to other parts of the model. However, the formation of stars in clusters does have the attraction that the intrinsic angular momentum that an interstellar cloud would almost be bound to possess can be taken up in the relative motion of stars within the cluster rather than in the spin of the stars themselves. Both the McCrea and Woolfson models lead to this conclusion and some better model of star formation in a cluster is likely to do the same.

Finally, one other aspect of solar spin should be mentioned: the 7° inclination of the solar spin axis to the net angular momentum vector of the planetary orbits. The figure of 7° seems

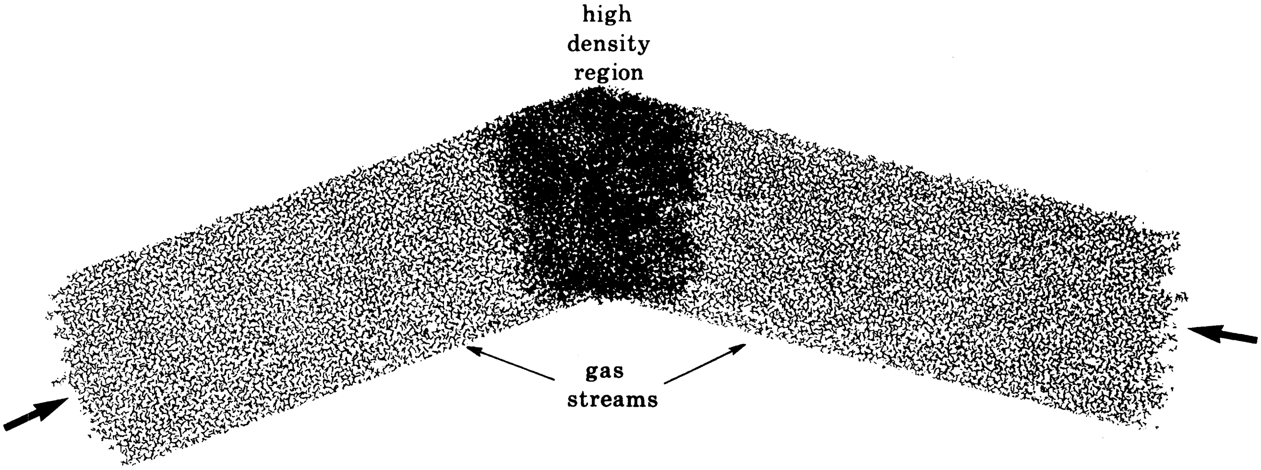

FIGURE 3. The collision of turbulent streams to give a high-density region. Rapid cooling of the heated material can lead to a supra-critical mass and collapse to form a star.

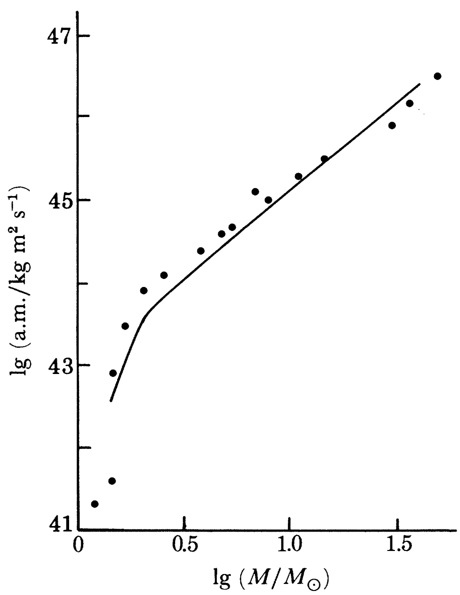

FIGURE 4. The relation between angular momentum and mass of stars. The full line is deduced from observation and the dots from the theory given by Woolfson (1979).

too high to be accommodated by any solar-nebula type of theory without introducing some *ad hoc* additional features. On the other hand it is also too low to be considered an accident; the probability of collinearity to this extent or better is less than 0.01.

In Woolfson's Capture Theory for the origin of the Solar System (Woolfson 1964), illustrated in figure 5, there is no relationship between the initial direction of the solar spin axis and the mean orbital plane of the planets. However, an intrinsic part of the Capture Theory is the development of a resisting medium around the Sun which modifies the planetary orbits (Dormand & Woolfson 1974, 1977). The decay of this medium is partly by absorption into the Sun which will tend to make its spin axis orthogonal to the mean orbital plane of the planets

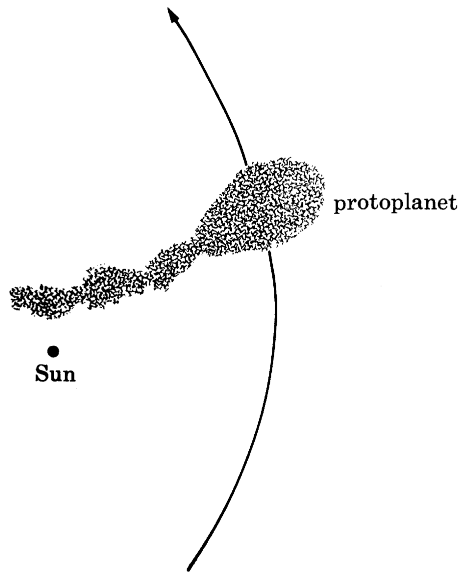

FIGURE 5. In the capture theory model protoplanets formed in a filament from a tidally disrupted protostar are captured by the Sun.

but without doing so precisely. It seems that this scenario can also be applied to McCrea's model or, indeed, to any theory, giving initially eccentric planetary orbits in the presence of a resisting medium.

3. Planetary spin

(*a*) *Planets from floccules*

McCrea's floccule theory (McCrea 1978) has very attractive features in relation to the origin of planetary spin and satellite formation. Planets are formed by aggregates of about 100 floccules which are captured by the newly formed Sun. However, McCrea calculated that for Jupiter and Saturn, for example, such aggregations would have 30–100 times the angular momentum that is actually observed. Such protoplanets could not condense at all and McCrea called on some ideas by Lyttleton (1960) on the fissional break-up of an incompressible rapidly rotating body. The first stage of such a process is shown in figure 6*a*. The body will divide into two parts with a mass ratio somewhat, but not much, greater than 8:1. When the body disrupts (figure 6*b*) most of the angular momentum will be taken up by the relative motion of the two parts about the centre of mass with the smaller part moving much more quickly. McCrea reasons that in the outer part of the system the smaller parts are moving fast enough to escape from the Solar System, leaving behind giant planets rotating in the way we now observe. From the model, rotation periods for Jupiter and Saturn are estimated as 8.6 and 12 hours compared with the observed periods of 9.8 and 10.2 hours respectively.

The proposed mechanism for the terrestrial planets is somewhat different in that it involves fission of only the solid part of the planetary condensations. McCrea suggests that, because of the higher escape velocity in the inner part of the system, the smaller part of the disrupted planet would be retained. He postulates that the pairs of planets Mercury–Venus and Earth–Mars could have been produced in this way. McCrea points out in favour of this hypothesis that the mean densities of the combined pairs of bodies are very similar. In addition the spin periods

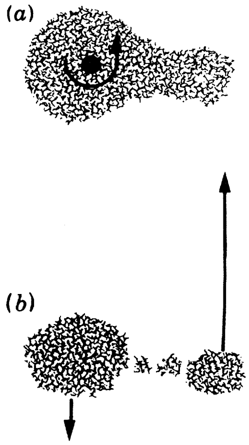

FIGURE 6. (*a*) A rapidly rotating protoplanet, as predicted by the floccule theory, evolving towards fission. (*b*) Break up of the protoplanet into unequal parts. The smaller part moves rapidly relative to the centre of mass. Droplets between the two parts may give a satellite family.

and inclinations of spin axes to orbital planes are very similar for the Earth and Mars as this scenario would suggest.

Another attractive by-product of the fission hypothesis is the formation of droplets between the retreating parts of the protoplanet as seen in figure 6*b* which would explain the presence of regular satellites for the major planets Jupiter, Saturn and Uranus although, apparently, not for Neptune, both of whose satellites are quite irregular.

(*b*) *Planets by accretion*

A high proportion of the work on planetary formation has been on accretion models starting with a solar nebula. It is generally agreed (see, for example, Gurevich & Lebedinskii 1950) that on a fairly short timescale, dust particles would settle and form a high-density disc. Goldreich & Ward (1973) showed that such a disc would be gravitationally unstable and would break up to form planetesimals with dimensions of a few kilometres or less. Safranov (1972) showed that the largest planetesimal in a region would grow by accretion much more rapidly than would smaller ones and so, by a runaway effect, one dominant body would evolve capable of absorbing all lesser bodies within its gravitational sphere of influence. The estimates of the timescales for the accretion of terrestrial planets was 10^7–10^8 years, but for the outer planets the timescales are uncomfortably large, up to 10^{10} years for Neptune.

A great deal of work has been done on the expected angular momentum of an accreted planet, for example by Guili (1968*a*, *b*), Lyttleton (1972), Harris (1977), Eneev & Kozlov (1981) and Schofield (1981). The very comprehensive analysis by Schofield shows that the outcome from any theory is very sensitive to the assumptions made. In a model he considered for formation of the Earth, if only those particles were accreted that actually intersected the planetary embryo then a proto-Earth would be formed in 1.1×10^5 years with negative angular momentum (retrograde spin). However, another assumption he tested was that the embryo captures virtually all the material coming within its sphere of influence because of inelastic collisions between streams of particles intersecting within that region. In this case growth to Earth mass is rapid (1.4×10^4 a) and the angular momentum is positive and high, 8.2×10^{35} kg m^2 s^{-1} compared with the observed values of 6×10^{33} kg m^2 s^{-1} for the spin of the Earth and

3.5×10^{34} kg m^2 s^{-1} for the Earth–Moon system. Schofield's diagram showing the net angular momentum contribution of particles as a function of their distance of closest approach is reproduced in figure 7.

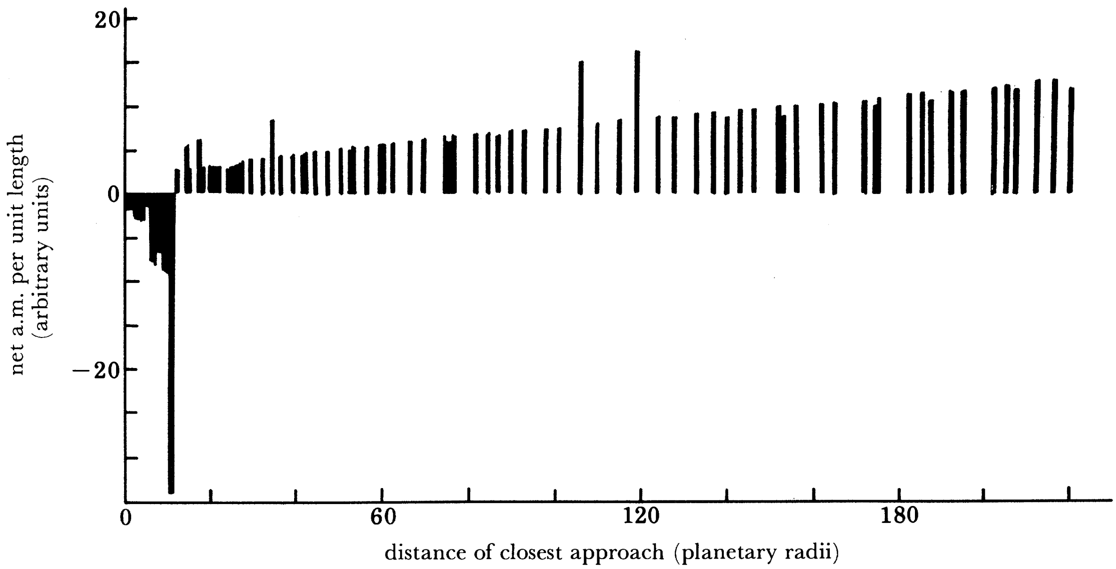

FIGURE 7. Particles approaching a planetary embryo impart angular momentum depending on their distance of closest approach (Schofield 1981).

Schofield considers that a realistic situation is somewhere between the two extremes detailed above. During the early part of the accretion, when the particle density is high, there would be frequent interactions among particles and all that passed through the sphere of influence would be captured. Later, as the particle density fell, so would the frequency of particle interactions so that only those actually intersecting the surface of the embryo would be captured. A detailed model based on this concept was examined by Schofield, who found that in 2.3×10^4 years the embryo would grow to terrestrial mass with angular momentum 5.3×10^{34} kg m^2 s^{-1}. However, in the final stages, the added angular momentum is negative, and the embryo, in a molten state, would pass through a rotationally unstable stage. It was suggested that the Earth–Moon system could have formed in this way. The Earth : Moon mass ratio, 81 : 1, is much higher than that given by Lyttleton (1960) of 8 : 1. In discussing this point Schofield noted a conclusion by O'Keefe (1969) that fission would be accompanied by high temperature associated with tidal dissipation, and it was inferred by Schofield that volatilization of some material would leave the Moon with only a fraction of its original mass.

Schofield also pointed out that the lack of a satellite and the slow rotation of Venus could be accommodated by the change of one of the adjustable parameters of his model. Thus if the final accretion stage which contributes negative angular momentum is expanded at the expense of the initial stage then the desired result for Venus may be obtained.

The extreme sensitivity of the estimated rotation rates to the values of arbitrary parameters of the model is somewhat unfortunate but this seems to be a characteristic of all accretion models.

(c) *The direct formation of protoplanets*

Some theories of the origin of the Solar System, in particular the Capture Theory, give diffuse gaseous protoplanets formed in highly eccentric orbits. Williams & Woolfson (1983) have considered the behaviour of such protoplanets in the first stages of their evolution. They took protoplanetary models as suggested by Schofield & Woolfson (1982) in orbits suggested by the Capture Theory model of Dormand & Woolfson (1977). The three planets considered were Jupiter, Saturn and Uranus. The general pattern of the evolution of the protoplanets is shown in figure 8. While the protoplanet is an extended body, prograde rotation is induced in it by solar tidal action. The angular momentum is concentrated in the tidal bulges, that on the Sun-facing side being of substantially greater size. When the protostar collapses it effectively decouples from the tidal influence of the Sun. The main tidal bulge evolves into a filament-like form and condensation within this leads to a family of regular satellites.

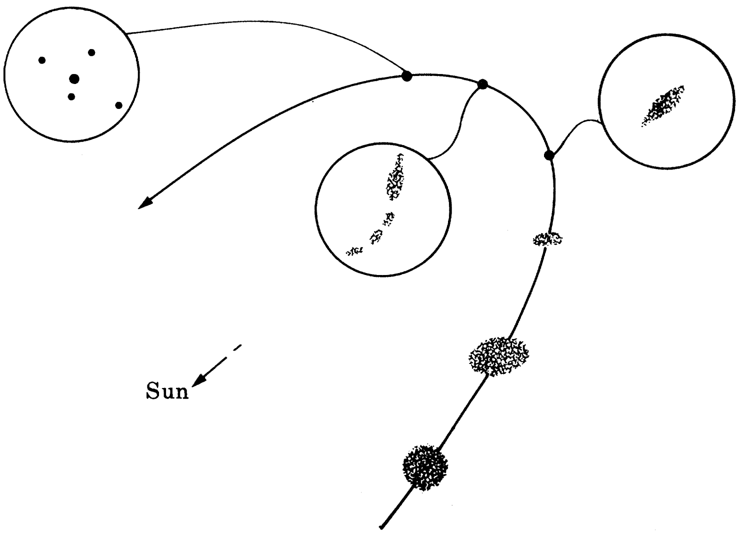

FIGURE 8. The evolution of a protoplanet on an elliptical orbit. The collapsing protoplanet eventually decouples from the solar tidal influence and protosatellites are formed in a tongue of material left behind. The circled views of the evolutionary stages are highly magnified.

There is a school of thought that claims that any proposed mechanism for giving planets in relation to the Sun should also give satellites in relation to planets. Thus Jeans (1928) wrote, 'any hypothesis which assigned different origins to the main system and the sub-systems would be condemned by its own artificiality'. More recently, Alfvén (1978) has stated, 'We should not try to make a theory of the origin of planets around the Sun but a *general theory of the formation of secondary bodies around a central body*. This theory should be applicable both to the formation of satellites and to the formation of planets.' However, it is worth noting that the mean angular momentum per unit mass of Galilean satellite in its orbit is some 20 times that for a unit mass at Jupiter's equator because of spin, a factor that is comfortably accommodated by a theory which separates the satellites from a rotating protoplanet, especially when the angular momentum is concentrated in the outer regions. The corresponding factor for the planet–Sun system is 20000 which is the nub of the problem for the solar-nebula theory. In the light of this pattern, to have a different mode of formation of the two types of system seems not to be artificial or unreasonable, and may even be inevitable. The results obtained by Williams & Woolfson by numerical analysis with their model are shown in table 2. The factor *f* in the second column is the ratio of the mean density of bulge material to that for the whole protoplanet. In determining the expected angular momentum in the original protoplanet it is assumed that the regular satellites are a 1 % residue of original gaseous bodies. One point which is interesting

TABLE 2. COMPARISON OF THEORETICAL INDUCED ANGULAR MOMENTA WITH EXPECTED INITIAL ANGULAR MOMENTA FOR JUPITER, SATURN AND URANUS

(It is assumed that the satellites are a 1 % residue of originally gaseous bodies. Units are kilograms times square metre per second.)

planet	theoretical a.m./*f*	observed spin a.m.	orbital a.m. of satellites	augmented a.m. of satellites	expected total initial a.m.
Jupiter	3.2×10^{39}	4.4×10^{38}	4.2×10^{36}	4.2×10^{38}	8.6×10^{38}
Saturn	6.2×10^{38}	9.1×10^{37}	9.5×10^{35}	9.5×10^{37}	1.9×10^{38}
Uranus	4.2×10^{37}	1.6×10^{36}	1.3×10^{34}	1.3×10^{36}	3.0×10^{36}

here is the near equality in the observed spin angular momentum and the augmented orbital angular momentum of the satellites. The conclusion which is drawn from table 2 is that about one half of the induced angular momentum contributes to planetary spin and the remainder to satellite formation. To explain the actual planetary spin rates, similar values of *f* (0.27 and 0.30) are required for Jupiter and Saturn but that for Uranus (0.07) is quite different. This suggests that the angular momentum predicted by the model for Uranus is too high.

(*d*) *Other features of planetary spins*

Dormand & Woolfson (1977) have shown that, according to the Capture Theory model, interactions between planets in the early Solar System should have been quite common. Thus the variety of directions of spin axes of the giant protoplanets could be due to mutual interactions. From the spin axis direction for Uranus it could be surmised that it underwent a particularly heavy interaction; its final angular momentum would then be a resultant of that acquired from solar interaction and that from the comparatively near passage of another planet. For example, the tidal effect of Saturn at a distance of 1.5 AU is not very different from that of the Sun at 20 AU.

Dormand & Woolfson postulated that the terrestrial planets were formed as a result of a collision between two large protoplanets in the region of the present asteroid belt. The larger planet acquired enough energy to leave the Solar System while the non-volatile residues of the smaller planet rounded off to give the dense terrestrial planets Mercury, Venus and the Earth. No useful speculation can be made about the spins of these planets; shear forces during the collision and tidal forces subsequently could be invoked to explain almost any observation.

As part of the scenario it has been suggested that Mars was originally a satellite of one of the colliding planets. The densities of terrestrial bodies and some larger satellites are shown in figure 9 in support of this suggestion. Connell & Woolfson (1983) have ascribed the hemispherical asymmetry of Mars, like that of the Moon, to abrasion by high-speed ejecta from the planetary collision of that face of the satellite turned towards its primary. This will give rise to a thinning of the crust and for Mars such features as the centre-of-mass centre of figure offset are well explained by this. If Mars as a satellite was in synchronous rotation about its

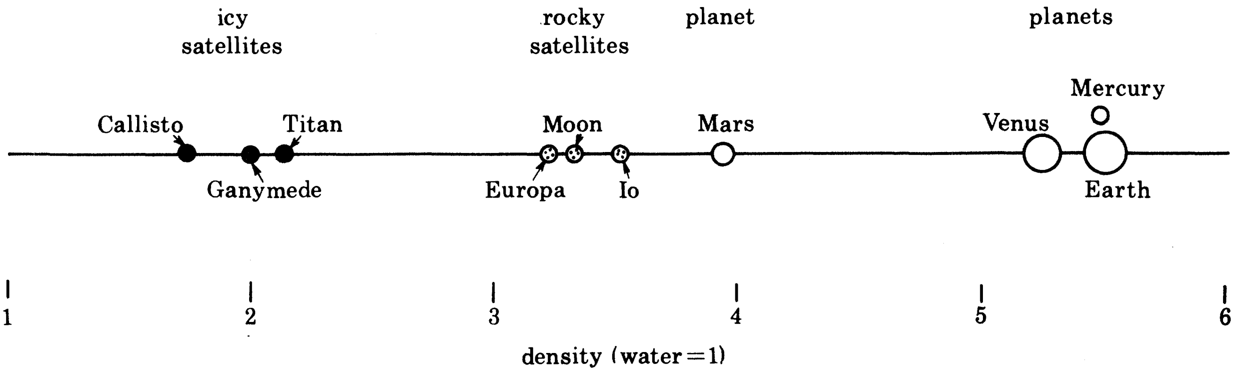

FIGURE 9. Terrestrial planets and some larger satellites strung out according to density.

primary then this mechanism would suggest that its spin axis should be contained in the plane of asymmetry; but it is actually at 55° to that plane.

Satellites formed by fast condensation in a filament would be expected to have molten material close to the surface early in its history and movement of the lithosphere over a low-viscosity mantle would readily take place. Connell & Woolfson have shown that the principal axis of maximum moment of inertia of the moment of inertia tensor calculated from a model of the major Martian surface features lies within a few degrees of the present spin axis. The idea of polar wander is not new; for example Runcorn (1983) has also suggested polar wander on the Moon to explain magnetic observations.

4. Conclusions

Unless the idea of a very high early magnetic dipole moment for the Sun is accepted then there appear to be great difficulties in explaining the slow rotation of a Sun produced from a solar nebula which is also to produce a planetary system. On the other hand, theories which form the Sun, and other late-type stars as part of a stellar cluster, by interactions in a turbulent interstellar cloud have no such difficulty and seem more promising from a theoretical point of view. However some subsidiary mechanism must then be found to account for planetary formation.

The floccule theory gives a good description of planetary formation and an explanation of their spin rates and satellite systems, marred only by the problem of floccule stability. Accretion models of planetary formation can, by a slight adjustment of parameters explain almost any planetary spin rate although such features as the different directions of spin axes are unexplained. The action of solar tidal forces on giant protoplanets in elliptical orbits seems to explain both the spin rates of these planets and the angular momentum associated with their regular satellite families. Interactions in the early Solar System can then account for the various directions of the planetary spin axes at least for the major planets.

References

Alfvén, H. 1978 In *The origin of the solar system* (ed. S. F. Dermott), pp. 19–40. Chichester: Wiley.
Alfvén, H. & Arrhenius, G. 1976 In *Evolution of the solar system*, §16. Washington: N.A.S.A.
Cameron, A. G. W. 1978 In *The origin of the solar system* (ed. S. F. Dermott), pp. 49–74. Chichester: Wiley.

Cameron, A. G. W. & Pine, M. R. 1973 *Icarus* **18**, 377–406.
Connell, A. J. & Woolfson, M. M. 1983 *Mon. Not. R. astr. Soc.* **204**, 1221–1230.
Dicke, R. H. & Goldenberg, H. M. 1967 *Phys. Rev. Lett.* **18**, 313–316.
Dormand, J. R. & Woolfson, M. M. 1974 *Proc. R. Soc. Lond.* A **340**, 349–365.
Dormand, J. R. & Woolfson, M. M. 1977 *Mon. Not. R astr. Soc.* **180**, 243–279.
Eneev, T. M. & Kozlov, N. N. 1981 *Astron. Vestn.* (U.S.S.R.), vol. 15, no. 3, 131–140.
Freeman, J. W. 1978 In *The origin of the solar system* (ed. S. F. Dermott), pp. 635–640. Chichester: Wiley.
Goldreich, P. & Ward, W. R. 1973 *Astrophys. J.* **183**, 1051–1061.
Guili, R. T. 1968*a* *Icarus* **8**, 301–323.
Guili, R. T. 1968*b* *Icarus* **9**, 186–190.
Gurevich, L. E. & Lebedinskii, A. I. 1950 *Dokl. Akad. Nauk SSSR* **74**, 905–907.
Harris, A. W. 1977 *Icarus* **31**, 168–174.
Jeans, J. 1928 In *Astronomy and cosmogony*, chap. XVI. Cambridge University Press.
Laplace, P. S. 1796 *Exposition du système du monde.* Paris: Imprimerie Cercle-Social.
Lynden-Bell, D. & Pringle, J. E. 1974 *Mon. Not. R. astr. Soc.* **168**, 603–637.
Lyttleton, R. A. 1960 *Mon. Not. R. astr. Soc.* **121**, 551–569.
Lyttleton, R. A. 1972 *Mon. Not. R. astr. Soc.* **158**, 463–483.
McCrea, W. H. 1960 *Proc. R. Soc. Lond.* A **256**, 245–266.
McCrea, W. H. 1978 In *The origin of the solar system* (ed. S. F. Dermott), pp. 75–110. Chichester: Wiley.
O'Keefe, J. A. 1969 *J. geophys. Res.* **74**, 2758–2767.
Runcorn, S. K. 1983 *Nature, Lond.* **304**, 589–596.
Safranov, V. S. 1972 *Evolution of the protoplanetary cloud and formation of the Earth and planets.* Jerusalem: Israel Program for Scientific Translations.
Schofield, N. 1981 *Mon. Not. R. astr. Soc.* **197**, 1031–1048.
Schofield, N. & Woolfson, M. M. 1982 *Mon. Not. R. astr. Soc.* **198**, 947–959.
Williams, S. & Woolfson, M. M. 1983 *Mon. Not. R. astr. Soc.* **204**, 853–863.
Woolfson, M. M. 1964 *Proc. R. Soc. Lond.* A **282**, 485–507.
Woolfson, M. M. 1979 *Phil. Trans. R. Soc. Lond.* A **291**, 219–252.

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